Invariant Slot Attention: Object Discovery with Slot-Centric Reference Frames

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Outline

- Authorship
- Background
- Method
- Experiments
- Conclusion

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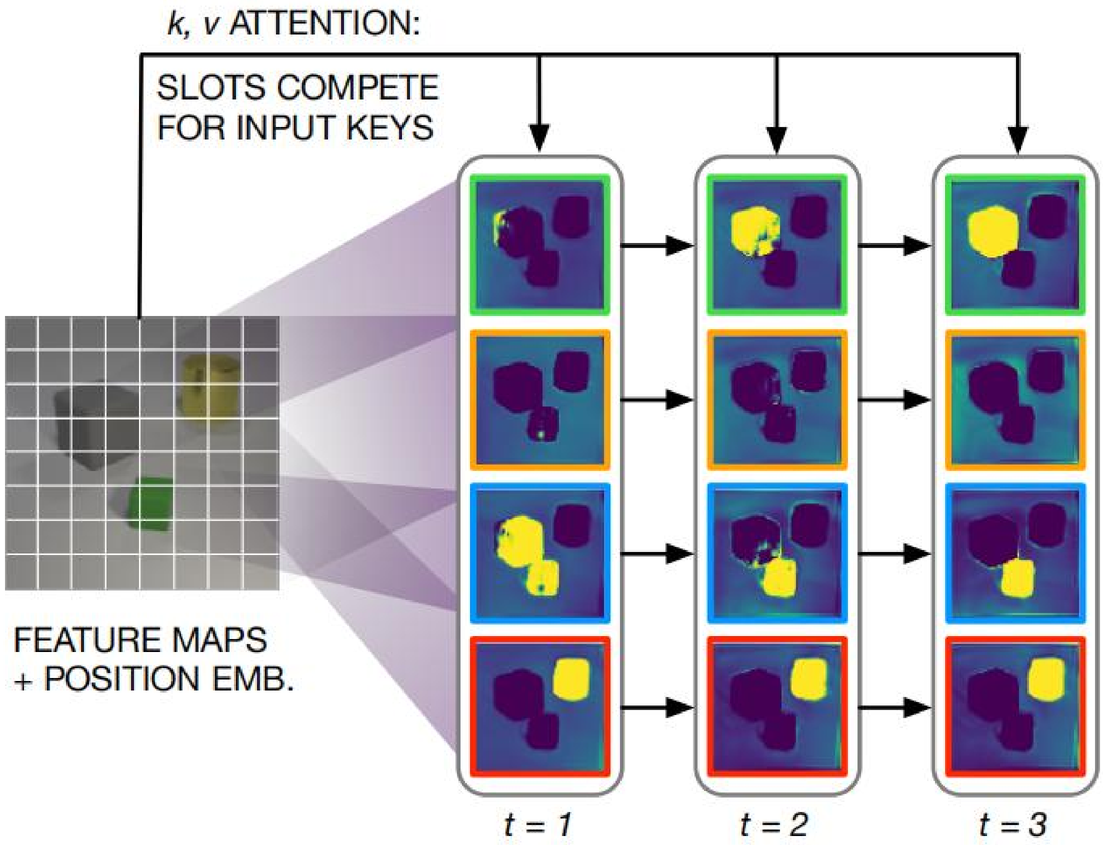
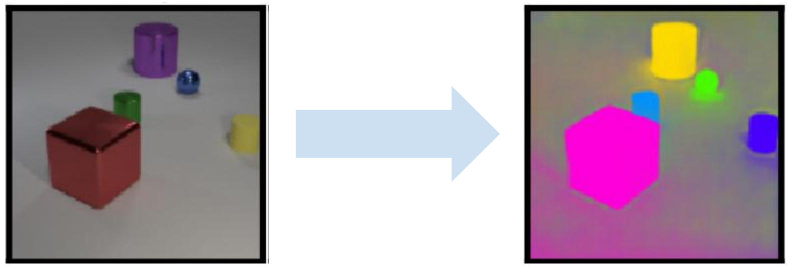
Slot Attention (1/4)

- Object-Centric Learning with Slot Attention (NeurIPS 2020)

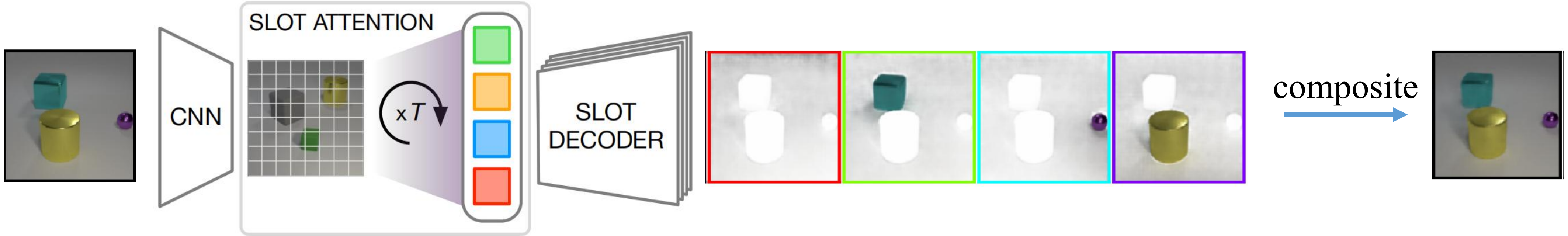


- Task: unsupervised object discovery, i.e., uncovering patterns that define objects and discriminates them against the background.
- More specifically, seperate an image to sets of pixels.



Slot Attention (2/4)

- Slots: Latent vector that describe a single object



- Encoder: Vanilla CNN, ResNet, etc. Output features of size $D_{inputs}*H_0*W_0$
- Slot attention module. Output k slots, each with size D_{slots}

1) Sampling: Sample all object latents (i.e., slots) from the same prior distribution to encourage representational uniformity across all slots. $\texttt{slots} \sim \mathcal{N}(\mu, \operatorname{diag}(\sigma)) \in \mathbb{R}^{K \times D_{\texttt{slots}}}$

2) Binding: Bound each slot to an object region via an attention mechanism.

3) Updating: Each slot gets updated by the bound object features to specialize for that object.

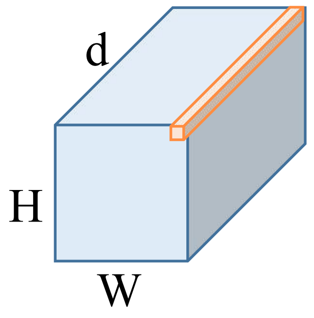
- Decoder: TransposeConvNets. Given a slot latent, broadcast it into an initial size (e.g, 8*8), then upsample to the original size. For each slot, output: $4*H*W$ (R, G, B, alpha). Alpha is a mask for compositing slots and determine the attribution of pixel.

Slot Attention (3/4)

- Primary Idea: Attention machanism
  - *Slots compete for explaining parts of the input*
  - Slots as queries
  - Features as keys&values (per input position, extracted by CNN, with positional embeddings)

add positional encoding

keys = $\mathcal{K}\Bigg($

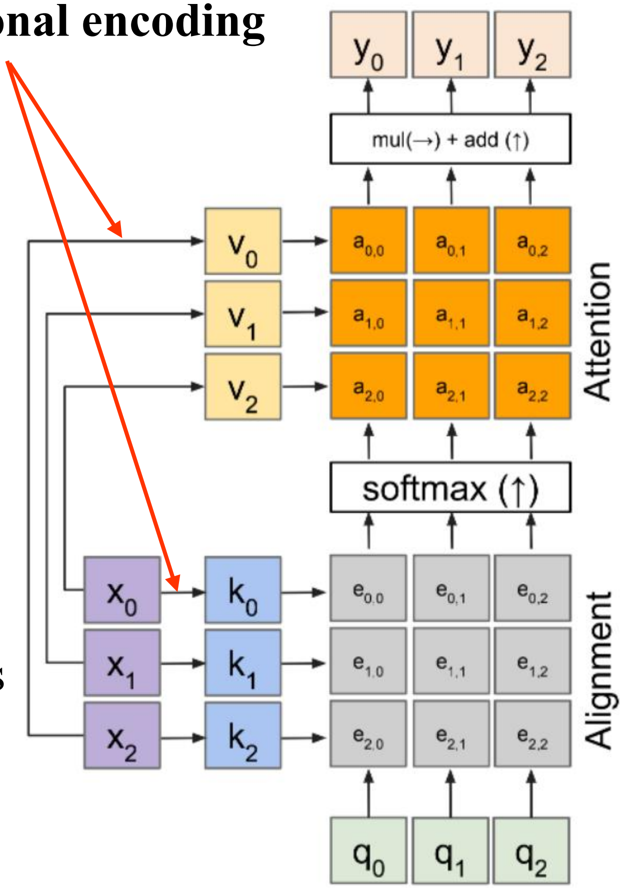


$$+ \; g\left(\begin{bmatrix} (-1,-1) & (-0.9,-1) & \cdots & (1,-1) \\ (-1,-0.9) & (-0.9,-0.9) & \cdots & (1,-0.9) \\ \vdots & \vdots & \ddots & \vdots \\ (-1,1) & (-0.9,1) & \cdots & (1,1) \end{bmatrix}\right)\Bigg)$$

input features

positional encoding

input features



slots

Slot Attention (4/4)

- Pseudo code for attention module

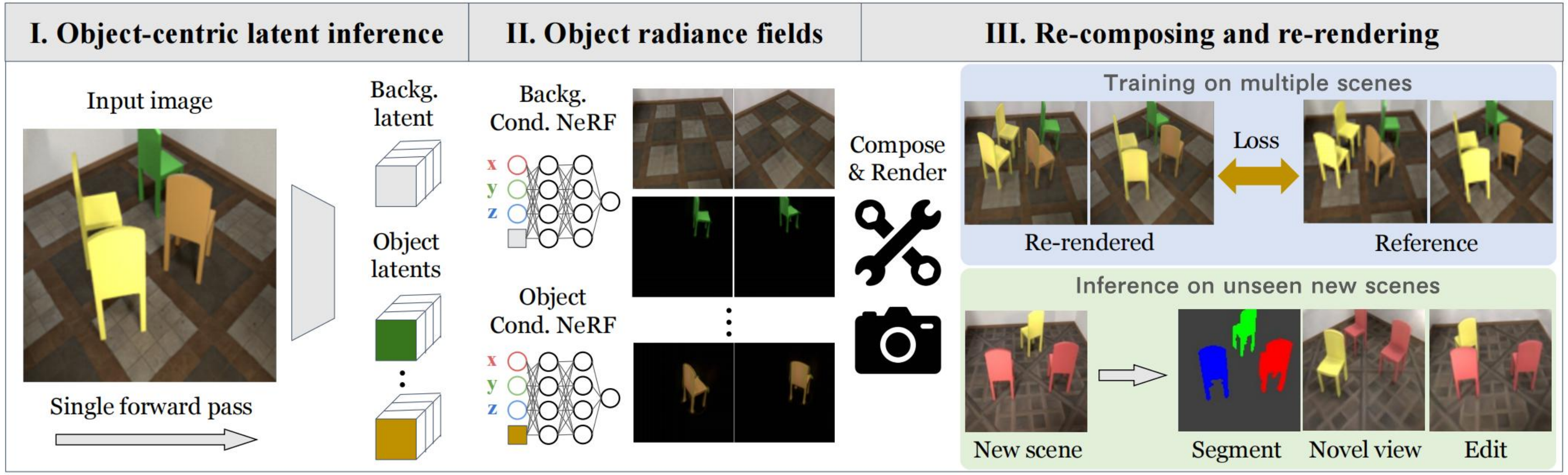
Input: $\texttt{inputs} \in \mathbb{R}^{N \times D_{\texttt{inputs}}}$, $\texttt{slots} \sim \mathcal{N}(\mu,\ \mathrm{diag}(\sigma)) \in \mathbb{R}^{K \times D_{\texttt{slots}}}$
Layer params: $k,\ q,\ v$: linear projections for attention; GRU; MLP; LayerNorm (x3)
    $\texttt{inputs} = \texttt{LayerNorm}(\texttt{inputs})$
    for $t = 0 \ldots T$
        $\texttt{slots\_prev} = \texttt{slots}$
        $\texttt{slots} = \texttt{LayerNorm}(\texttt{slots})$
        $\texttt{attn} = \texttt{Softmax}\left(\frac{1}{\sqrt{D}} k(\texttt{inputs}) \cdot q(\texttt{slots})^T,\ \texttt{axis='slots'}\right)$ # norm. over slots
        $\texttt{updates} = \texttt{WeightedMean}(\texttt{weights=attn} + \epsilon,\ \texttt{values=}v(\texttt{inputs}))$ # aggregate
        $\texttt{slots} = \texttt{GRU}(\texttt{state=slots\_prev},\ \texttt{inputs=updates})$ # GRU update (per slot)
        $\texttt{slots} \mathrel{+}= \texttt{MLP}(\texttt{LayerNorm}(\texttt{slots}))$ # optional residual MLP (per slot)
    return slots

LayerNorm: Normalize feature instead of batch

GRU: Gated Recurrent Unit (RNN with gates, similar to LSTM)

Slot Attention in 3D (1/1)

- Unsupervised Discovery of Object Radiance Fields (ICLR 2022)

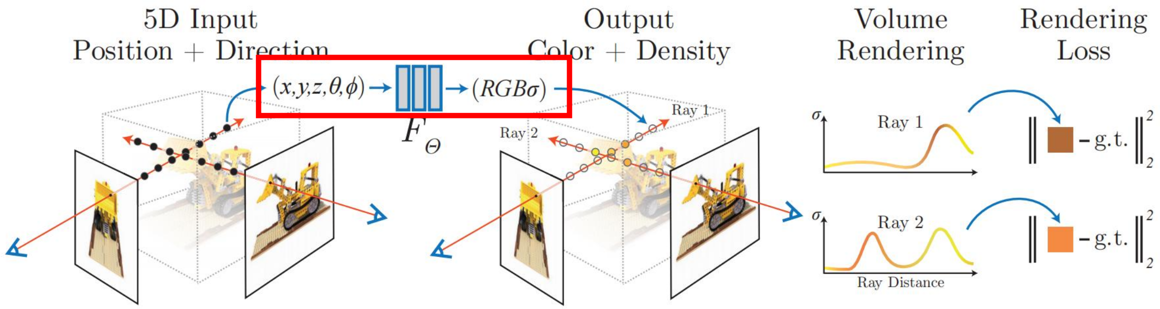


Single image NeRF for scene editing & synthesis

- Given a single reference image, extract *slot latents* for scence segmentation, decompostion, etc.
- Condition NeRF on the slot latent
- Trained on reconstruction loss

Slot Attention in 3D (2/2)

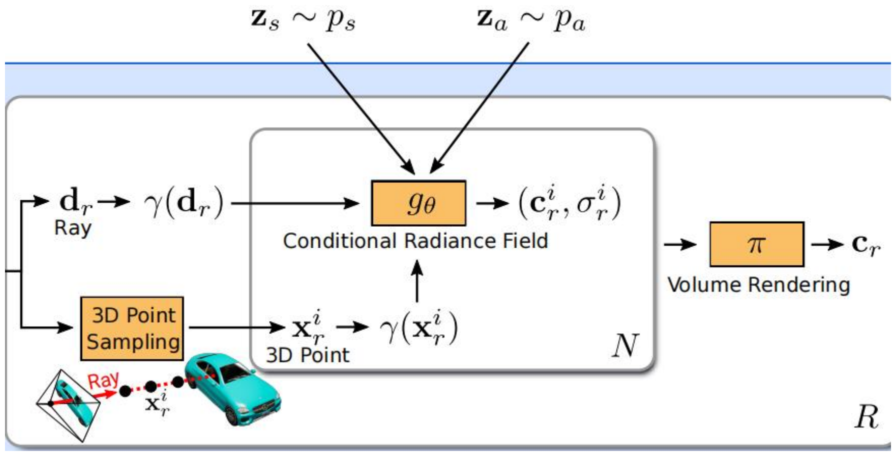
- NeRF Overview



- Volume rendering

$$C(\mathbf{r}) = \int_{t_n}^{t_f} T(t)\sigma(\mathbf{r}(t))\mathbf{c}(\mathbf{r}(t), \mathbf{d})dt\,, \text{ where } T(t) = \exp\left(-\int_{t_n}^{t} \sigma(\mathbf{r}(s))ds\right).$$

- Conditional NeRF



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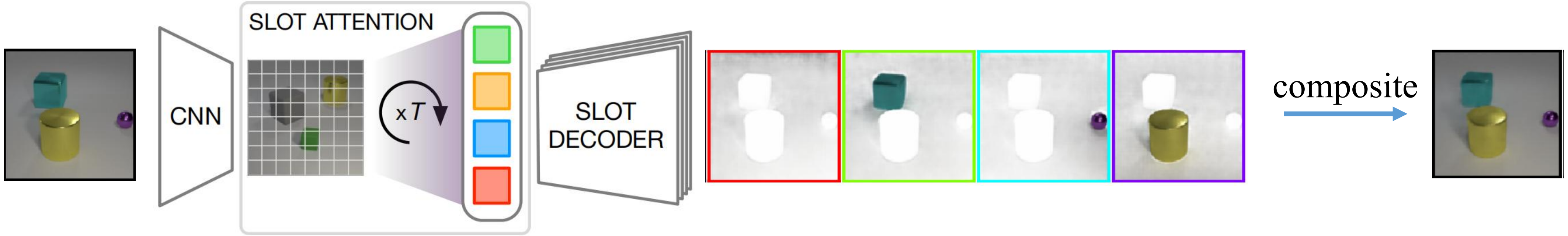
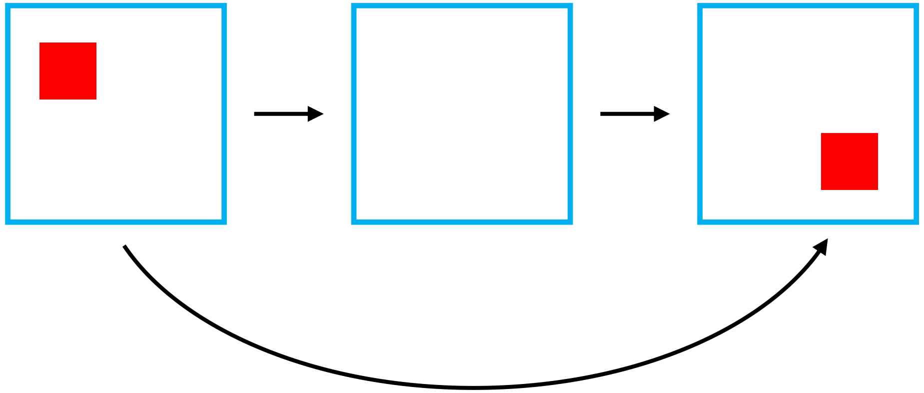
Slot Attention: Conclusion



One sentence summary: use slots to explain objects.

Problems of slot attention

- Slot is a mixture of object information ➡ How to disentangle "slot"?



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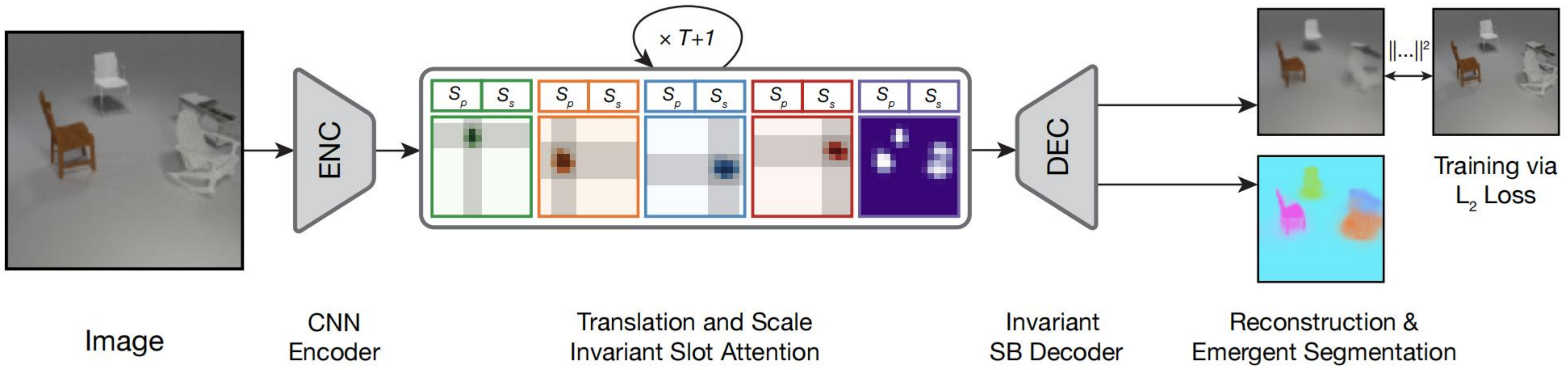
Method: Overview (1/6)



- Disentangle *slot's* appearance with its position, scale, and rotation.
- Specifically, learn the position, scale, and rotation (S_p, S_s, S_r) of each slot

$$S_p \in \mathcal{R}^2, S_s \in \mathcal{R}^2, S_r \in [0, \pi]$$

- As a result, slot feature is invariant to position, scale, and rotation.

Method: Invariance to Translation and Scaling (2/6)

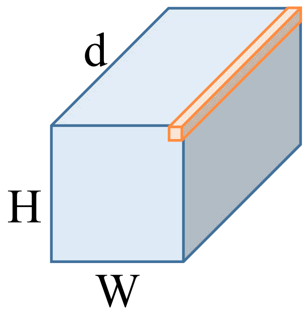
- In original slot attention, we add positional encoding to each input feature
- Relative positional encoding for each slot (K slots in total)

$$\mathrm{rel\_grid}^k = (\mathrm{abs\_grid} - S_p^k) \;/\; S_s^k \qquad k \in \{1, ..., K\}$$

$$\begin{bmatrix} (-1,-1) & (-0.9,-1) & \cdots & (1,-1) \\ (-1,-0.9) & (-0.9,-0.9) & \cdots & (1,-0.9) \\ \vdots & \vdots & \ddots & \vdots \\ (-1,1) & (-0.9,1) & \cdots & (1,1) \end{bmatrix} \xrightarrow{S_p^0 = (0.5, 0.5),\, S_s^0 = (1,1)} \begin{bmatrix} (-1.5,-1.5) & (-1.4,-1.5) & \cdots & (0.5,-1.5) \\ (-1.5,-1.4) & (-1.4,-1.4) & \cdots & (0.5,-1.4) \\ \vdots & \vdots & \ddots & \vdots \\ (-1.5,0.5) & (-1.4,0.5) & \cdots & (0.5,0.5) \end{bmatrix}$$

Uniform positional encoding — Relative positional encoding for slot 0

$\mathrm{keys}^0 = \mathcal{K}\Bigg($



$$+\; g\left(\begin{bmatrix} (-1.5,-1.5) & (-1.4,-1.5) & \cdots & (0.5,-1.5) \\ (-1.5,-1.4) & (-1.4,-1.4) & \cdots & (0.5,-1.4) \\ \vdots & \vdots & \ddots & \vdots \\ (-1.5,0.5) & (-1.4,0.5) & \cdots & (0.5,0.5) \end{bmatrix}\right)\Bigg)$$

input features — relative positional encoding

Method: Invariance to Translation and Scaling (3/6)

- Obtain slot keys and values

$$\text{keys}^k = f\left(\mathcal{K}(\text{inputs}) + g(\text{rel\_grid}^k)\right)$$

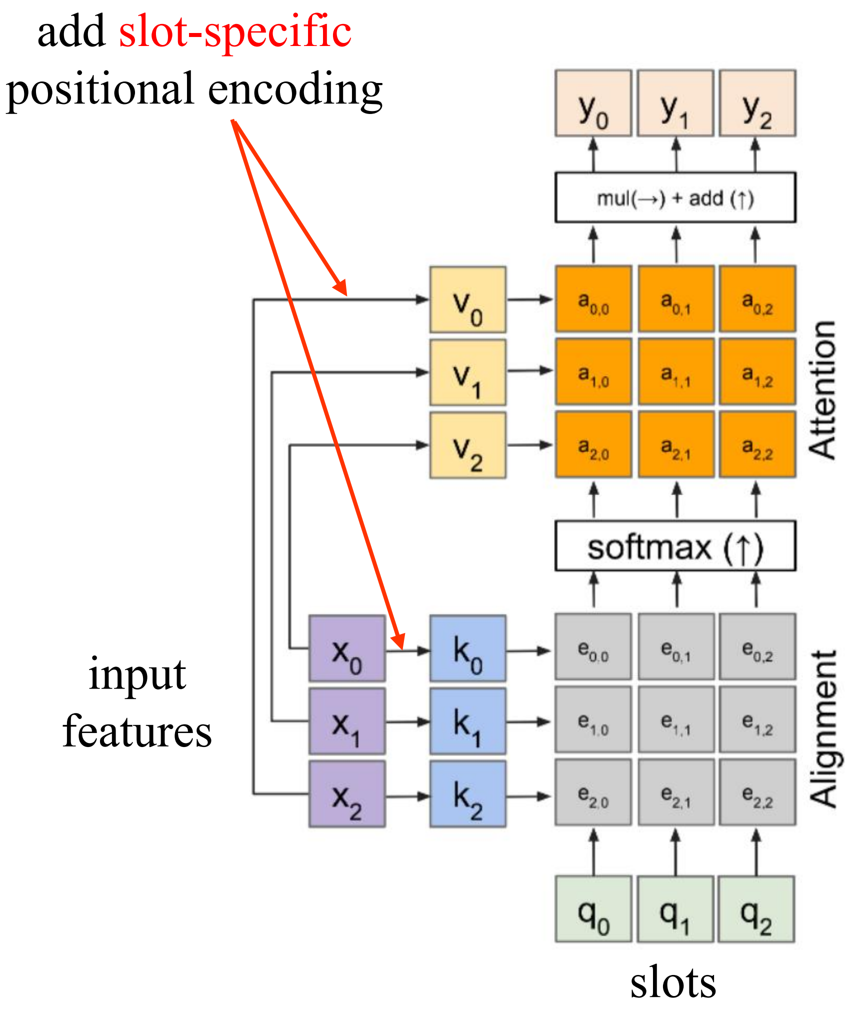
$$\text{values}^k = f\left(\mathcal{V}(\text{inputs}) + g(\text{rel\_grid}^k)\right)$$

- Obtain attention and updates
- Update slots by GRU
- Update slots' position and scale latent

$$S_p = \frac{\sum_{n=1}^{N} \text{attn}_n * \text{abs\_grid}_n}{\sum_{n=1}^{N} \text{attn}_n}$$

$$S_s = \sqrt{\frac{\sum_{n=1}^{N} (\text{attn}_n + \epsilon) * (\text{abs\_grid}_n - S_p)^2}{\sum_{n=1}^{N} (\text{attn}_n + \epsilon)}}$$

Intuitive explanation: move slot to the place with higher attention



Method: Invariance to Rotation (4/6)

- Encode rotation (S_r)
  - Heuristic: the orientation of a slot is given by the axis with the highest variation

$$v_1^k, v_2^k = \text{WeightedPCA}(\text{abs\_grid}, \text{attn}^k)$$

eigenvector of the covariance matrix

$$\tilde{v}_1^k, \tilde{v}_2^k = \text{post-process}\left(v_1^k, v_2^k\right) ,$$

$$S_r^k = \begin{bmatrix} | & | \\ \tilde{v}_1^k & \tilde{v}_2^k \\ | & | \end{bmatrix} .$$

$$\text{rel\_grid} = [S_r^{-1}(\text{abs\_grid} - S_p)] \,/\, (S_s \times \delta)$$

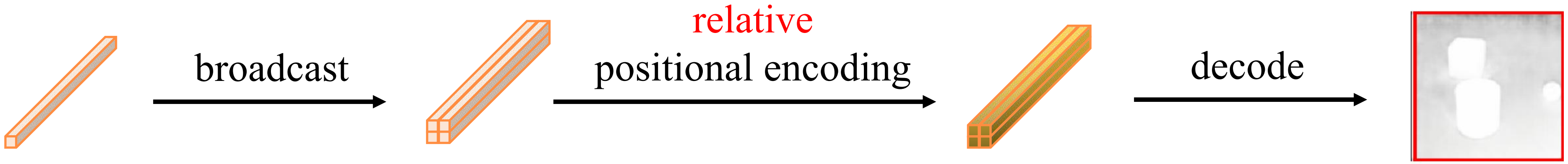
- Author's claim: Not effective enough
- My opinion: Unreasonable to encode rotation in 3D by 2D rotation matrix

Method: Decoding (5/6)

- Decode each slot
  - Calculate *rel_grid* for current slot
  - Spatially broadcast slot with relative positional encoding
  - Decode the RGB value and alpha mask (an image per slot)

$$(R, G, B, \alpha) = D_\phi(\mathrm{SB} + h(\mathrm{rel\_grid})) \qquad \mathrm{rel\_grid}^k = (\mathrm{abs\_grid} - S_p^k) \;/\; S_s^k$$

- Alpha composite the images



Method (6/6)

Algorithm 1 Translation, Rotation, and Scaling Invariant Slot Attention.

Inputs: inputs $\in \mathbb{R}^{N \times D_{inputs}}$, abs_grid $\in \mathbb{R}^{N \times 2}$, slots $\in \mathbb{R}^{K \times D_{slots}}$, Slot positions, $S_p \in \mathbb{R}^{K \times 2}$, Slot rotations, $S_r \in \mathbb{R}^{K \times 2 \times 2}$, Slot scales, $S_s \in \mathbb{R}^{K \times 2}$, T iterations, small ϵ.
Data: Encoders f, g, k, v, q, parameters of LayerNorms, MLP and GRU, δ.
Outputs: slots $\in \mathbb{R}^{K \times D_{slot}}$, $S_p \in \mathbb{R}^{K \times 2}$, $S_r \in \mathbb{R}^{K \times 2 \times 2}$, $S_s \in \mathbb{R}^{K \times 2}$.

inputs $=$ LayerNorm(inputs)
for $t = 1$ **to** $T + 1$ **do**
    slots_prev $=$ slots
    slots $=$ LayerNorm(slots)

    # Computes relative grids per slot, and associated key, value embeddings.
    rel_grid $= [S_r^{-1}(\text{abs\_grid} - S_p)] \;/\; (S_s \times \delta)$
    keys $= f\,(k(\text{inputs}) + g(\text{rel\_grid}))$
    values $= f\,(v(\text{inputs}) + g(\text{rel\_grid}))$

    # Inverted dot production attention.
    attn $= \text{softmax}(\frac{1}{\sqrt{K}}\text{keys} * q(\text{slots})^T, \text{axis} = \text{“slots”})$
    updates $= \text{WeightedMean}(\text{weights} = \text{attn}, \text{values} = \text{values})$
    attn $/= \text{Sum}(\text{attn}, \text{axis} = \text{“inputs”})$

    # Updates S_p, S_s and slots.
    $S_p = \text{WeightedMean}(\text{weights} = \text{attn}, \text{values} = \text{abs\_grid})$
    $S_r = \text{Symmetrize}(\text{WeightedPCAAnalytical}(\text{inputs} = \text{abs\_grid} - S_p, \text{weights} = \text{attn})$
    $S_s = \sqrt{\text{WeightedMean}(\text{weights} = \text{attn} + \epsilon, \text{values} = [S_r^{-1}(\text{abs\_grid} - S_p)]^2)}$
    if $t < T + 1$ **then**
        slots $= \text{GRU}(\text{state} = \text{slots\_prev}, \text{inputs} = \text{updates})$
        slots $+= \text{MLP}(\text{LayerNorm}(\text{slots}))$
    end if
end for

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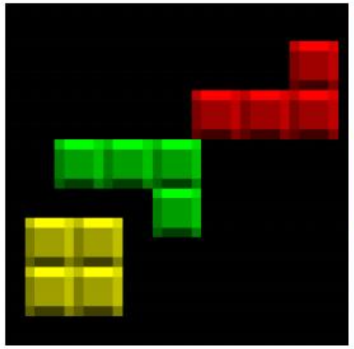
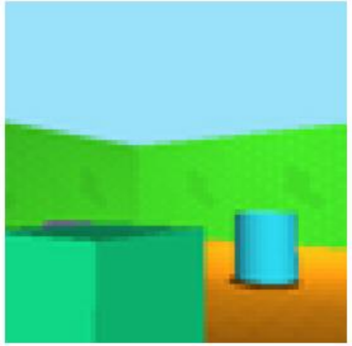
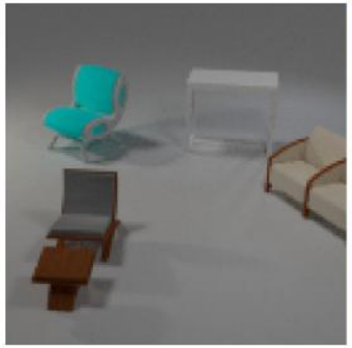
Eeperiments (1/6)

- Datasets
  - Tetrominos
  - Objects Room
  - MultiShapeNet
  - CLEVR
  - CLEVR Tex
  - Waymo Open (real-world, only for qualitative)
- Evaluation Protocol
  - Qualitative
  - Quantitative (FG-ARI)

Eeperiments (2/6)

- Rand Index: Calculate the similarity between two partitions of a set

Given a set of n elements $S = \{o_1, \ldots, o_n\}$ and two partitions of S to compare, $X = \{X_1, \ldots, X_r\}$, a partition of S into r subsets, and $Y = \{Y_1, \ldots, Y_s\}$, a partition of S into s subsets, define the following:

- a, the number of pairs of elements in S that are in the **same** subset in X and in the **same** subset in Y
- b, the number of pairs of elements in S that are in **different** subsets in X and in **different** subsets in Y
- c, the number of pairs of elements in S that are in the **same** subset in X and in **different** subsets in Y
- d, the number of pairs of elements in S that are in **different** subsets in X and in the **same** subset in Y

The Rand index, R, is:[1][2]

$$R = \frac{a+b}{a+b+c+d} = \frac{a+b}{\binom{n}{2}}$$

- Adjusted Rand Index:

$X \diagdown Y$	Y_1	Y_2	$\cdots$	Y_s	sums
X_1	n_{11}	n_{12}	$\cdots$	n_{1s}	a_1
X_2	n_{21}	n_{22}	$\cdots$	n_{2s}	a_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
X_r	n_{r1}	n_{r2}	$\cdots$	n_{rs}	a_r
sums	b_1	b_2	$\cdots$	b_s	

Definition [edit]

The original Adjusted Rand Index using the Permutation Model is

$$ARI = \frac{\sum_{ij} \binom{n_{ij}}{2} - \left[\sum_i \binom{a_i}{2} \sum_j \binom{b_j}{2}\right] / \binom{n}{2}}{\frac{1}{2}\left[\sum_i \binom{a_i}{2} + \sum_j \binom{b_j}{2}\right] - \left[\sum_i \binom{a_i}{2} \sum_j \binom{b_j}{2}\right] / \binom{n}{2}}$$

Experiments: Quantitative (3/6)

Table 1. CLEVRTex FG-ARI(%) results on the test set, CAMO set (objects and backgrounds blend together) and OOD set (novel textures). Prior results taken from (Karazija et al., 2021) use 3 random seeds, we use 10 random seeds. FG-ARI is reported in %. For MSE please see the Table 8. (CNN) refers to models using a 4-layer CNN backbone, while (ResNet) models use a ResNet-34.

Method	**Main**	**CAMO**	**OOD**
SPACE	17.5 ±4.1	10.6 ±2.1	12.7 ±3.4
DTI	79.9 ±1.4	72.9 ±1.9	73.7 ±1.0
AST-Seg-B3-CT	94.8 ±0.5	87.3 ±3.8	83.1 ±0.8
SA (CNN)	54.5 ±1.6	53.0 ±1.6	54.2 ±2.6
ISA-T (CNN)	66.8 ±5.7	65.0 ±4.9	65.1 ±4.8
ISA-TS (CNN)	78.8 ±3.9	72.9 ±3.5	73.2 ±3.1
SA (ResNet)	91.3 ±2.7	84.9 ±2.9	81.4 ±1.4
ISA-T (ResNet)	87.4 ±6.6	79.0 ±5.9	78.6 ±4.9
ISA-TS (ResNet)	92.9 ±0.4	86.2 ±0.8	84.4 ±0.8

- Rotation invariance does not bring consistent improvement

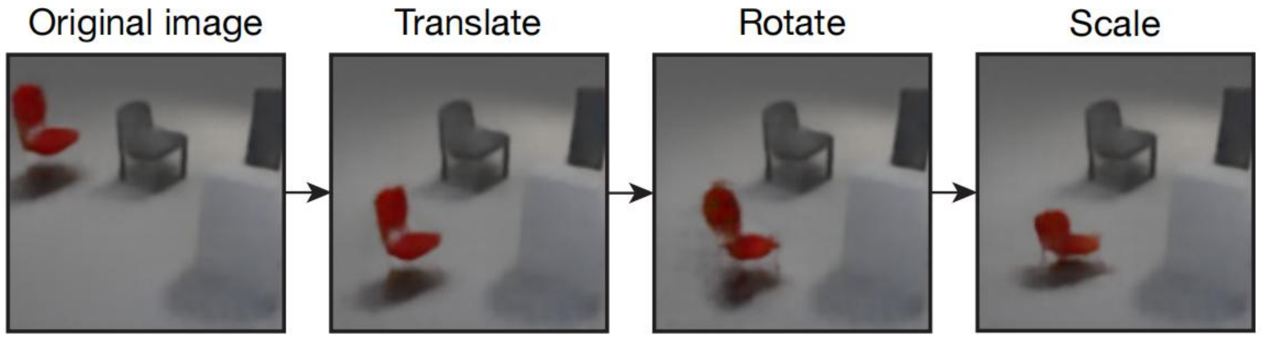
Table 2. **Rotation invariance**: Comparing ISA-TS against ISA-TSR in various benchmarks. Objects Room results are ARIs whereas all others are FG-ARIs. Remaining benchmarks evaluations are in the appendix Table 4.

	(FG-)ARI ↑	
Dataset	ISA-TS	ISA-TSR
Objects Room (w/ bg) Val.	85.5 ±6.6	84.3 ±4.6
CLEVR	98.9 ±0.2	98.0 ±0.9
MultiShapeNet		
- All Data	69.8 ±1.1	77.7 ±5.5
- Four Objects	86.5 ±1.1	80.7 ±6.4
CLEVRTex (CNN)	78.8 ±3.9	79.6 ±5.5
CLEVRTex (ResNet)	92.9 ±0.4	93.3 ±0.7

Experiments: Qualitative (4/6)

- ISA-T improves OOD robustness

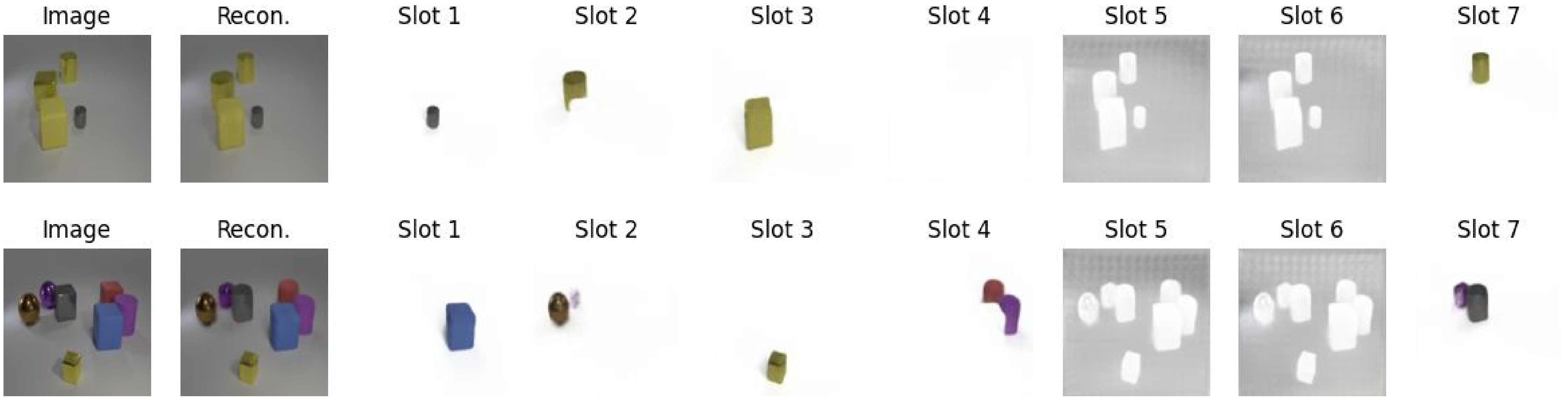
- Qualitative results on MultiShapeNet



- Qualitative results on Waymo open

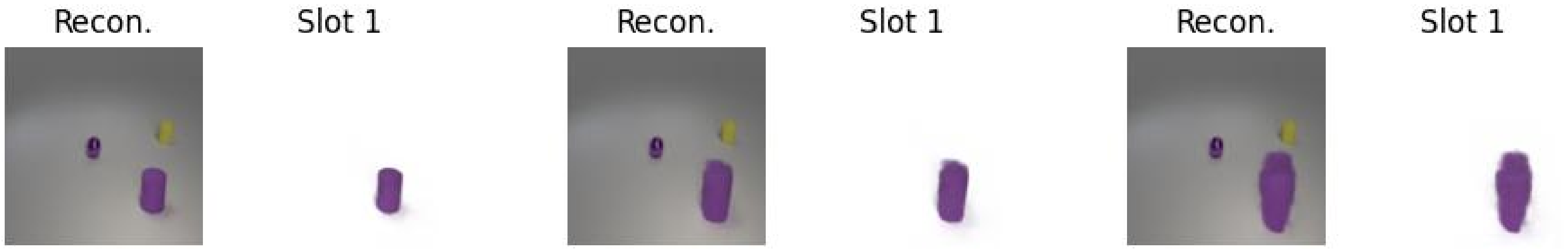
Experiments: Reproduction (5/6)

- Cannot guarantee one slot represent one object
- No explicit control on background (multiple background slot)
- Cannot handle occulusion (general problem for 2D methods)
- Undesirable reconstruction result on edges (e.g, tend to smooth sharp edges)



Experiments: Reproduction (6/6)

- Results for translation and scaling



translate   translate, scale*1.5   translate, scale*2

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Conclusion

Pros

- The authors proposed invariant slot attention, a novel approach for obtaining object-centric representations from 2D single image

Cons

- Position and scale are defined in the image plane, while objects position is 3D
- This approach seems only work for 2D

Future research direction

- 3D object-centric representations from a single image.

Thanks for listening!