

#### **EDICT: Exact Diffusion Inversion via Coupled Transformations**

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## Outline

- Authorship
- Background
- Architecture
- Experiments



#### **Generative Models**



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$$\mathbb{E}\left[-\log p_{\theta}(\mathbf{x}_{0})\right] \leq \mathbb{E}_{q}\left[-\log \frac{p_{\theta}(\mathbf{x}_{0:T})}{q(\mathbf{x}_{1:T}|\mathbf{x}_{0})}\right] = \mathbb{E}_{q}\left[-\log p(\mathbf{x}_{T}) - \sum_{t \geq 1}\log \frac{p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t})}{q(\mathbf{x}_{t}|\mathbf{x}_{t-1})}\right] =: L$$
$$= \mathbb{E}_{q}\left[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{T}|\mathbf{x}_{0}) \parallel p(\mathbf{x}_{T}))}_{L_{T}} + \sum_{t>1}\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t}))}_{L_{t-1}} \underbrace{-\log p_{\theta}(\mathbf{x}_{0}|\mathbf{x}_{1})}_{L_{0}}\right]$$



 $\mathbb{E}_{q}\left[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{T}|\mathbf{x}_{0}) \parallel p(\mathbf{x}_{T}))}_{L_{T}} + \sum_{t>1}\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t}))}_{L_{t-1}} \underbrace{-\log p_{\theta}(\mathbf{x}_{0}|\mathbf{x}_{1})}_{L_{0}}\right]$   $q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) = \mathcal{N}(\mathbf{x}_{t-1}; \tilde{\boldsymbol{\mu}}_{t}(\mathbf{x}_{t},\mathbf{x}_{0}), \tilde{\beta}_{t}\mathbf{I}),$ where  $\tilde{\boldsymbol{\mu}}_{t}(\mathbf{x}_{t},\mathbf{x}_{0}) \coloneqq \frac{\sqrt{\bar{\alpha}_{t-1}}\beta_{t}}{1-\bar{\alpha}_{t}}\mathbf{x}_{0} + \frac{\sqrt{\alpha_{t}}(1-\bar{\alpha}_{t-1})}{1-\bar{\alpha}_{t}}\mathbf{x}_{t} \text{ and } \tilde{\beta}_{t} \coloneqq \frac{1-\bar{\alpha}_{t-1}}{1-\bar{\alpha}_{t}}\beta_{t}$   $L_{t-1} - C = \mathbb{E}_{\mathbf{x}_{0},\epsilon} \left[\frac{1}{2\sigma_{t}^{2}} \left\| \tilde{\boldsymbol{\mu}}_{t}\left(\mathbf{x}_{t}(\mathbf{x}_{0},\epsilon), \frac{1}{\sqrt{\bar{\alpha}_{t}}}(\mathbf{x}_{t}(\mathbf{x}_{0},\epsilon) - \sqrt{1-\bar{\alpha}_{t}}\epsilon)\right) - \boldsymbol{\mu}_{\theta}(\mathbf{x}_{t}(\mathbf{x}_{0},\epsilon), t) \right\|^{2} \right] \tag{9}$ 

$$= \mathbb{E}_{\mathbf{x}_{0},\boldsymbol{\epsilon}} \left[ \frac{1}{2\sigma_{t}^{2}} \left\| \frac{1}{\sqrt{\alpha_{t}}} \left( \mathbf{x}_{t}(\mathbf{x}_{0},\boldsymbol{\epsilon}) - \frac{\beta_{t}}{\sqrt{1-\bar{\alpha}_{t}}} \boldsymbol{\epsilon} \right) - \boldsymbol{\mu}_{\theta}(\mathbf{x}_{t}(\mathbf{x}_{0},\boldsymbol{\epsilon}),t) \right\|^{2} \right]$$
(10)



$$\mathbb{E}_{q}\left[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{T}|\mathbf{x}_{0}) \parallel p(\mathbf{x}_{T}))}_{L_{T}} + \sum_{t>1}\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t}))}_{L_{t-1}} \underbrace{-\log p_{\theta}(\mathbf{x}_{0}|\mathbf{x}_{1})}_{L_{0}}\right]$$
$$\mathbb{E}_{\mathbf{x}_{0},\boldsymbol{\epsilon}}\left[\frac{\beta_{t}^{2}}{2\sigma_{t}^{2}\alpha_{t}(1-\bar{\alpha}_{t})} \parallel \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta}(\sqrt{\bar{\alpha}_{t}}\mathbf{x}_{0} + \sqrt{1-\bar{\alpha}_{t}}\boldsymbol{\epsilon},t) \parallel^{2}\right]$$

Algorithm 1 Training	Algorithm 2 Sampling
1: repeat 2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$ 3: $t \sim \text{Uniform}(\{1, \dots, T\})$ 4: $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \mathbf{I})$ 5: Take gradient descent step on $\nabla_{\theta} \  \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta}(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t}\boldsymbol{\epsilon}, t) \ ^2$ 6: until converged	1: $\mathbf{x}_T \sim \mathcal{N}(0, \mathbf{I})$ 2: for $t = T,, 1$ do 3: $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ if $t > 1$ , else $\mathbf{z} = 0$ 4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left( \mathbf{x}_t - \frac{1 - \alpha_t}{\sqrt{1 - \overline{\alpha}_t}} \boldsymbol{\epsilon}_{\theta}(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$ 5: end for 6: return $\mathbf{x}_0$



$$\mathbb{E}_{q}\left[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{T}|\mathbf{x}_{0}) \parallel p(\mathbf{x}_{T}))}_{L_{T}} + \sum_{t>1}\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t}))}_{L_{t-1}} \underbrace{-\log p_{\theta}(\mathbf{x}_{0}|\mathbf{x}_{1})}_{L_{0}}\right]$$
$$\mathbb{E}_{\mathbf{x}_{0},\boldsymbol{\epsilon}}\left[\frac{\beta_{t}^{2}}{2\sigma_{t}^{2}\alpha_{t}(1-\bar{\alpha}_{t})} \parallel \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta}(\sqrt{\bar{\alpha}_{t}}\mathbf{x}_{0} + \sqrt{1-\bar{\alpha}_{t}}\boldsymbol{\epsilon},t) \parallel^{2}\right]$$

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Improve DDPM because:

- 1. T = 1000 is too big. Accelerate!
- 2. Retraining is disturbing. Same objective!
- 3. Results are stochastic. Deterministic sampling!



Improve DDPM with:

- 1. Non-Markov chain
- 2. Same marginals, different joints
- 3. Modeling the uncertainty with  $\sigma_t$



Review objectives of DDPM:

$$\mathbb{E}_{q}\left[\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{T}|\mathbf{x}_{0}) \parallel p(\mathbf{x}_{T}))}_{L_{T}} + \sum_{t>1}\underbrace{D_{\mathrm{KL}}(q(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) \parallel p_{\theta}(\mathbf{x}_{t-1}|\mathbf{x}_{t}))}_{L_{t-1}} \underbrace{-\log p_{\theta}(\mathbf{x}_{0}|\mathbf{x}_{1})}_{L_{0}}\right]$$





Review the special property of DDPM:

$$q(\boldsymbol{x}_t|\boldsymbol{x}_0) := \mathcal{N}(\boldsymbol{x}_t; \sqrt{\alpha_t}\boldsymbol{x}_0, (1-\alpha_t)\boldsymbol{I});$$

$$\begin{aligned} \mathbf{q}_{\sigma}(\mathbf{x}_{t-1}|\mathbf{x}_{t},\mathbf{x}_{0}) &= \mathcal{N}\left(\sqrt{\alpha_{t-1}}\mathbf{x}_{0} + \sqrt{1-\alpha_{t-1}-\sigma_{t}^{2}} \cdot \frac{\mathbf{x}_{t} - \sqrt{\alpha_{t}}\mathbf{x}_{0}}{\sqrt{1-\alpha_{t}}}, \sigma_{t}^{2}\mathbf{I}\right) \\ q_{\sigma}(\mathbf{x}_{t}|\mathbf{x}_{0}) &= \mathcal{N}(\mathbf{x}_{t};\sqrt{\alpha_{t}}\mathbf{x}_{0}, (1-\alpha_{t})\mathbf{I}); \end{aligned}$$



#### Objective:

$$J_{\sigma}(\epsilon_{\theta}) := \mathbb{E}_{\boldsymbol{x}_{0:T} \sim q_{\sigma}(\boldsymbol{x}_{0:T})} \left[ \log q_{\sigma}(\boldsymbol{x}_{1:T} | \boldsymbol{x}_{0}) - \log p_{\theta}(\boldsymbol{x}_{0:T}) \right]$$

$$= \mathbb{E}_{\boldsymbol{x}_{0:T} \sim q_{\sigma}(\boldsymbol{x}_{0:T})} \left[ \log q_{\sigma}(\boldsymbol{x}_{T} | \boldsymbol{x}_{0}) + \sum_{t=2}^{T} \log q_{\sigma}(\boldsymbol{x}_{t-1} | \boldsymbol{x}_{t}, \boldsymbol{x}_{0}) - \sum_{t=1}^{T} \log p_{\theta}^{(t)}(\boldsymbol{x}_{t-1} | \boldsymbol{x}_{t}) - \log p_{\theta}(\boldsymbol{x}_{T}) \right]$$

$$(11)$$





$$\boldsymbol{x}_{t-1} = \sqrt{\alpha_{t-1}} \underbrace{\left(\frac{\boldsymbol{x}_t - \sqrt{1 - \alpha_t} \epsilon_{\theta}^{(t)}(\boldsymbol{x}_t)}{\sqrt{\alpha_t}}\right)}_{\text{"predicted } \boldsymbol{x}_0\text{"}} + \underbrace{\sqrt{1 - \alpha_{t-1} - \sigma_t^2} \cdot \epsilon_{\theta}^{(t)}(\boldsymbol{x}_t)}_{\text{"direction pointing to } \boldsymbol{x}_t\text{"}} + \underbrace{\sigma_t \epsilon_t}_{\text{random noise}}$$



What does  $\sigma_t$  represent:

$$\begin{aligned} \mathbf{x}_{t-1} &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} \boldsymbol{\epsilon}_{t-1} \\ &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \boldsymbol{\epsilon}_t + \sigma_t \boldsymbol{\epsilon} \\ &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}} + \sigma_t \boldsymbol{\epsilon} \\ &q_{\sigma}(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_{t-1}; \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}}, \sigma_t^2 \mathbf{I}) \end{aligned}$$



What does  $\sigma_t$  represent:

$$\begin{split} \mathbf{x}_{t-1} &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} \boldsymbol{\epsilon}_{t-1} \\ &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \boldsymbol{\epsilon}_t + \sigma_t \boldsymbol{\epsilon} \\ &= \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}} + \sigma_t \boldsymbol{\epsilon} \\ &q_{\sigma}(\mathbf{x}_{t-1} | \mathbf{x}_t, \mathbf{x}_0) = \mathcal{N}(\mathbf{x}_{t-1}; \sqrt{\bar{\alpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_{t-1}} - \sigma_t^2 \frac{\mathbf{x}_t - \sqrt{\bar{\alpha}_t} \mathbf{x}_0}{\sqrt{1 - \bar{\alpha}_t}}, \sigma_t^2 \mathbf{I}) \end{split}$$

$$\begin{aligned} \sigma_t &= 0 \Rightarrow \epsilon_{t-1} = \epsilon_t \\ \sigma_t &= \sqrt{1 - \bar{\alpha}_{t-1}} \Rightarrow \epsilon_{t-1}, \epsilon_t \text{ i. i. d} \end{aligned}$$



# **GAN Inversion**





# **GAN Inversion**





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#### **Extract Diffusion Inversion**

- The generative process in DDIMs is defined in a non-Markovian manner, which results in a deterministic denoising process.
- DDIM can also be used for inversion, deterministically noising an image to obtain the initial noise vector.

• DDIM inversion is unstable in many cases.\*

<sup>\*</sup> Amir Hertz, et al. Prompt-to-prompt image editing with cross attention control. arXiv preprint arXiv:2208.01626, 2022.







"A couple standing together holding Wii controllers next to a building."









Recon. MSE = 0.011 Recon. MSE = 0.050 Recon. MSE = 0.044



#### **Extract Diffusion Inversion**

• DDIM inversion relies on the local linear assumption.

$$x_t = \frac{x_{t-1} - b_t \epsilon(x_t, t)}{a_t} \approx \frac{x_{t-1} - b_t \epsilon(x_{t-1}, t)}{a_t}$$

• Recall:

$$egin{aligned} \mathbf{x}_{t-1} &= \sqrt{ar{lpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - ar{lpha}_{t-1}} oldsymbol{\epsilon}_{t-1} \ &= \sqrt{ar{lpha}_{t-1}} \mathbf{x}_0 + \sqrt{1 - ar{lpha}_{t-1}} - \sigma_t^2 oldsymbol{\epsilon}_t + \sigma_t oldsymbol{\epsilon} \end{aligned}$$



# Affine Coupling Layers

• Widely used in flow-based models.

$$z = [z_a, z_b]$$

$$z'_a = \Psi(z_b)z_a + \psi(z_b)$$

$$z_a = (z'_a - \psi(z_b))/\Psi(z_b)$$



• Observe the similarity of

$$z'_a = \Psi(z_b)z_a + \psi(z_b)$$

and

$$x_{t-1} \coloneqq a_t x_t + b_t \epsilon(x_t, t)$$

, replace second  $x_t$  with  $y_t$ 

$$x_{t-1} = a_t x_t + b_t \epsilon(y_t, t)$$



$$x_{t-1} = a_t x_t + b_t \epsilon(y_t, t)$$

is invertible

$$x_t = (x_{t-1} - b_t \cdot \epsilon(y_t, t))/a_t$$

Define the updating function:  $x_{t-1} = a_t x_t + b_t \cdot \epsilon(y_t, t)$  $y_{t-1} = a_t y_t + b_t \cdot \epsilon(x_{t-1}, t)$ 



$$x_{t-1} = a_t x_t + b_t \cdot \epsilon(y_t, t)$$
$$y_{t-1} = a_t y_t + b_t \cdot \epsilon(x_{t-1}, t)$$

is invertible

$$y_t = (y_{t-1} - b_t \cdot \epsilon(x_{t-1}, t))/a_t$$
$$x_t = (x_{t-1} - b_t \cdot \epsilon(y_t, t))/a_t$$



The gap between defined procedure and DDIM depends of the gap of  $x_t, x_{t-1}$  and  $y_t$ .  $x_{t-1} = a_t x_t + b_t \cdot \epsilon(y_t, t)$ 

$$y_{t-1} = a_t y_t + b_t \cdot \epsilon(x_{t-1}, t)$$
  
should be  $x_t$  should be  $x_t$ 



#### **EDICT-Stabilization**

#### Directly using it is unsatisfactory due to error accumulation.





#### **EDICT-Stabilization**

Propose mixing layers:

$$x' = px + (1-p)y, \ 0 \le p \le 1$$

to stretch them closer.

$$\begin{aligned} x_t^{inter} &= a_t \cdot x_t + b_t \cdot \epsilon(y_t, t) \\ y_t^{inter} &= a_t \cdot y_t + b_t \cdot \epsilon(x_t^{inter}, t) \\ x_{t-1} &= p \cdot x_t^{inter} + (1-p) \cdot y_t^{inter} \\ y_{t-1} &= p \cdot y_t^{inter} + (1-p) \cdot x_{t-1} \end{aligned}$$





#### **EDICT-Stabilization**





## **EDICT-Application**

While EDICT can theoretically operate on either pixel-based or latent-diffusion models, we present the latter case in this work.

Noising w/ condition  $C_{base}$ , denoising w/ condition  $C_{target}$ .



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#### Reconstruction

Method	LDM AE	EDICT (UC)	EDICT (C)	DDIM (UC)	DDIM (C)
50 Steps	0.015	0.015	0.015	0.030	0.420
100 Steps	0.015	0.015	0.015	0.027	0.471
200 Steps	0.015	0.015	0.015	0.023	0.497

COCO Reconstruction Error (MSE)

Table 1. Mean-square reconstruction error for COCO-val using the first listed prompt as conditioning with G = 7. The latent diffusion model autoencoder (LDM AE) is the lower bound on reconstruction error. Using half precision increases 50-step EDICT (C) MSE by 6%. More step values are in the Supplementary.



# Editing



#### Real Image

"A lake"



"A giraffe in .."



"A car stuck in.."



"A castle overlooking .."





"A fountain in.."



"A red chair"

- ".. at the Grand Canyon"



".. on a field of grass"



".. covered in snow in the mountains















".. standing alone giving a thumbs-up"



"A waterfall in the mountains"





















Original Description "A cat"→ Image edit using prompt: "A ferret"



Original Description "A stone church"→ Image edit using prompt: "A stone church in wildflowers"



#### Conclusion

- A method to promise the invertibility of diffusion steps.
- Performs well both on conditional and unconditional cases.
- Less changes on irrelative regions with editing.
- No FID scores.
- Insufficient comparison.

# Thanks for your listening!